ISSN 2347 - 3983
Volume 8. No. 6, June 2020
International Journal of Emerging Trends in Engineering Research
Available Online at http://www.warse.org/IJETER/static/pdf/file/ijeter97862020.pdf
https://doi.org/10.30534/ijeter/2020/97862020

The Criterion and Evaluation of Effectiveness of Image Comparison in Correlation-Extreme Navigation Systems of Mobile Robots

Olha Liashko[1], Valentuyna Klindukhova[2],NataliiaYeromina[3], TetianaKaradobrii[4], OlenaBairamova[5], Antonina Dorosheva[6]

[1]Department of Higher and Applied Mathematics, State University of Infrastructure and Technologies, Ukraine, Kyiv, olha_liashko@ukr.net

[2] Department of Higher and Applied Mathematics, State University of Infrastructure and Technologies, Ukraine, Kyiv, klinduhova@ukr.net

[3]Department of Electronic Computers, Kharkiv National University of Radio Electronics, Ukraine, Kharkiv, nataliia.yeromina@nure.ua

[4]Kiliya Transport College of the State University of Infrastructure and Technologies, Ukraine, Izmail, karadobri1961@ukr.net

[5]Department of Natural, Mathematical and Engineering Sciences Disciplines, the Danube Faculty of Marine Transport of the State University of Infrastructure and Technologies, Ukraine, Izmail, df_duit@ukr.net

[6] Department of Navigation and Technical Systems Operation in Water Transport, the Danube Faculty of Marine Transport of the State University of Infrastructure and Technologies, Ukraine, Izmail, dorosheva.21.12@gmail.com

ABSTRACT

In this article we present the results of theoretical and practical evaluation of effectiveness of decision function (DF) generation, as a result of image comparison, in the navigation systems of the mobile robots (MR), based on the tracking and comparison method. It has been suggested to use the probability measure as a criterion for the evaluation of the effectiveness of DF generation.The results of the theoretical evaluation of effectiveness of DF generation have been obtained, the results of the statistical modeling of DF generation process have been displayed.The necessity of RI generation and image comparison in strong scales has been demonstrated.

Key words: navigation systems, mobile robots, the probability of decision function generation, strong scale.

1. INTRODUCTION

The navigation process of MR, equipped with the correlation-extreme navigation system (CENS), has the stochastic nature considering its dependence on the multiplicity of random factors.The part of these factors directly relate to the system itself, which induces the necessity to minimize their influence on the system operation.Firstly this refers to the system of generation ofthe current image (CI), the used reference images (RI), which are formed by some means or other, as well as to the image comparison method that is defined by the type of the algorithms in the corresponding scale.Basically, these factors define the characteristics of the navigation system [1, 2, 3], which necessitates the search of the most optimal solutions with regard to the effectiveness of DF generation.

1.1 Problem analysis

In the articles [4, 5, 6, 7] the results of the research, aimed at the optimal RI generation in different scales, are presented, the general approaches to the DF generation in CENS are analyzed, the algorithms for image comparison, including those formed with consideration to external factors and peculiarities of MR usage, are suggested. However, the results of the research, aimed at obtaining the effective solutions from the system perspective regarding the RI generation and the choice of the algorithm for image comparison in the corresponding scales, are missing.

The object of this article is to define the most rational way of generation of DF for MR from the system perspective.

2. MAIN MATERIAL

2.1 The criterion and general approaches to the theoretical evaluation of image comparison methods

When evaluating the effectiveness of different methods of image comparison, the RI, used in the CENS, is commonly represented as the Gaussian static ergodic random field. Concurrently, the RI, which has been prepared in advance using the relevant databases is deterministic. Hence, it is expedient to evaluate the effectiveness P_c of the algorithms of different types and to compare the theoretical evaluation results with the ones received with the help of statistical modeling.
The proof of the problem of the theoretical evaluation of the sought probability P_c will be received using the assumption that:
1)the positioning area of MR is known, that is the $N_1 \times N_2$ -matrix of RI $\left[a_{ij}\right]$ is given;

2) the grid nodes of RI and CI (current image) coincide;
3) RI is represented as the matrix with the dimensions $(M_1<N_1, M_2<N_2)$;
4) CI is described with the help of additive model $\hat{z}_{ij}=\hat{a}_{ij}+\hat{n}_{ij}$, $i\hat{I}\overline{1,N_1}$, $j\hat{I}\overline{1,N_2}$, , where $n_{ij}\in N(0,1)$;
5) the rotation of the RI with respect to the CI is absent.

Let us use the following notions for proof of the problem:
1. b – of DF;
2. The dimensions of the matrix of DF values is – $(N_1-M_1+1)\times(N_2-M_2+1)$;
3. The total quantity of the compared elements of CI is – $K=(N_1-M_1+1)(N_2-M_2+1)$.

The population of DF values will be represented with the help of the random vector $\mathbf{b}=(b_1,\ldots,b_K)$.
Let us display the matrix in lines and mark the number of the source fragment, which corresponds to the CI, as p.
Let us assume that A – is the event, which corresponds to the proper match of the RI and CI, and B_p – is the event, when the number of the source fragment p is equal.
In case of obtaining the decision on the DF minimum, the conditional probability $P(A/B_p)$ can be written as the following probability of event:

$$b_p<b_i,\ i\in\overline{1,K},\ i\neq p. \tag{1}$$

Assuming the K -dimensional probability density function W_{Kb} of the vector $\mathbf{b}$ is known,we shall introduce the random vector $\mathbf{t}$, which is linked to the vector $\mathbf{b}$ by the following set of equations:

$$\begin{cases} t_p=b_p, \\ t_i=b_p-b_i,\ i\in\overline{1,K},\ i\neq p. \end{cases} \tag{2}$$

Representing the set of equations (2) with respect to b_i, we will obtain:

$$\begin{cases} b_p=t_p, \\ b_i=t_p-t_i,\ i\in\overline{1,K},\ i\neq p. \end{cases} \tag{3}$$

Let us take into consideration the following:
If the random vectors $\mathbf{b}$, $\mathbf{t}$ are linked by the dependence $t_i=f_i(b_1,\ldots,b_K)$, $i\in\overline{1,K}$, and $W_{K\mathbf{b}}(x_1,\ldots,x_K)$, is set, then

$$W_{K\mathbf{t}}(y_1,\ldots,y_K)= =\sum_{i=1}^{r}W_{K\mathbf{b}}(x_1,\ldots,x_K)\left(\frac{\partial(x_{1i},\ldots,x_{Ki})}{\partial(y_1,\ldots,y_K)}\right), \tag{4}$$

where x_{ki} – is the i-th branch, opposite to (2)transformation of variables.

$\left(\frac{\partial(x_{1i},\ldots,x_{Ki})}{\partial(y_1,\ldots,y_K)}\right)$ – is the Jacobian of transformation of variables for i-thbranch.

In this context the reciprocal transformation is defined by the equation (3) and has one branch.
The module ofJacobian of transformation of variables equals to 1, therefore the relation (4) can be presented as follows:

$$W_{K\mathbf{t}}(y_1,\ldots,y_K)= =W_{K\mathbf{b}}(y_p-y_1,\ldots,y_p-y_{p-1},y_p,y_p-y_{p+1},\ldots,y_p-y_K). \tag{5}$$

Let us denote the K -dimensional probability density function of the vector $\mathbf{t}$ $F_{K\mathbf{t}}$.
As a result the following continued equality is received:

$$=\int_{-\infty}^{0}dy_1 L\int_{-\infty}^{0}dy_{p-1}\int_{-\infty}^{0}dy_p\int_{-\infty}^{0}dy_{p+1}L\int_{-\infty}^{0}dy_K\,W_{Kt}(y_1,\ldots,y_K)\ . \tag{6}$$

Lets plug (5) into (6) and when changing the variables we will obtain:

$$P\left(\frac{A}{B_p}\right)= =\int_{y_p}^{\infty}dy_1 L\int_{y_p}^{\infty}dy_{p-1}\int_{-\infty}^{\infty}dy_p\int_{y_p}^{\infty}dy_{p+1}L\int_{y_p}^{\infty}dy_K\,W_{Kb}(y_1,..,y_K). \tag{7}$$

If the decision is made based on the maximum DF value, then:

$$P\left(\frac{A}{B_p}\right)= =\int_{-\infty}^{y_p}dy_1 L\int_{-\infty}^{y_p}dy_{p-1}\int_{-\infty}^{\infty}dy_p\int_{-\infty}^{y_p}dy_{p+1}L\int_{-\infty}^{y_p}dy_K\,W_{Kb}(y_1,\ldots,y_K). \tag{8}$$

If the probabilities $P(B_p)$ of events B_p are given in advance, then, based on the relation of total probability, we will receive:

$$P_c=P(A)=\sum_{p=1}^{R}P(B_p)P(A/B_p). \tag{9}$$

<u>The quadratic difference algorithm</u> is characterized by the following type of DF:

$$b_k=\sum_{i=1}^{M}\left(\frac{\hat{y}_i^k-\hat{e}_i}{\sigma_i^k}\right)^2, \tag{10}$$

where $\hat{\mathbf{z}}^k=(\hat{z}_1^k,\ldots,\hat{z}_M^k)$, $\hat{\mathbf{e}}=(\hat{e}_1,\ldots,\hat{e}_M)$ – are the vector representations of $\mathbf{k}$ -th fragment of CI and RI in case of the representation of their matrix as lines;

$\boldsymbol{\sigma}^k = \left(\sigma_1^k,\ldots,\sigma_M^k\right)$ – is the matrix representation of $\mathbf{k}$-th fragment of the matrix of the standard deviation of system channels' interior noises.

For the additive model of interaction of image signaling component and channel noise, the $\mathbf{k}$-th fragment of CI can be written as:

$$\hat{y}^k = \hat{\mathbf{a}}^k + \hat{\mathbf{n}}^k,\ k \in \overline{1,K}, \tag{11}$$

where $\hat{\mathbf{a}}^k = \left(\hat{a}_1^k,\ldots,\hat{a}_M^k\right)$ – is the vector representation of noise-free $\mathbf{k}$-th fragment of CI;

$\hat{\mathbf{n}}^k = \left(\hat{n}_1^k,\ldots,\hat{n}_M^k\right)$ –is the vector of internal noise of channels, used for generation of $\mathbf{k}$-th fragment of CI.

Let us draw vectors

$$\xi^k = \left(\xi_1^k,\ldots,\xi_M^k\right),\ \xi_i^k = \left(\hat{a}_i^k - \hat{e}_i + \hat{n}_i^k\right)\big/\sigma_i^k,\ k \in \overline{1,K}.$$

It is evident that $\xi_i^k \in \mathbf{N}\left(\left(\hat{a}_i^k - \hat{e}_i\right)\big/\sigma_i^k, 1\right)$.

Let us denote by $\chi^2(m, M)$ the noncentral χ^2-distribution with M degree of freedom and noncentrality parameter m, then the random variable will be written as:

$$b_k \in \chi^2\left(m^k, M\right),$$

where

$$m^k = \sum_{i=1}^{M}\left(\frac{\hat{a}_i^k - \hat{e}_i}{\sigma_i^k}\right)^2. \tag{12}$$

The value m^k in physical meaning is the weight-average noise-to-signal ratio for $\mathbf{k}$-th fragment of CI, if signal represents the energy of differences of RI and $\mathbf{k}$-th fragment of CI.

The one-dimensional density function for components b_k of vector $\mathbf{b}$ is defined by the expression,

$$W_{b_k}(y) = \frac{\exp\left[-\left(y + m^k/2\right)\right]}{2^{M/2}} y^{M/2-1} \sum_{j=0}^{\infty} \frac{\left(m^k y/4\right)^j}{j!\Gamma\left(j + M/2\right)},$$
$$y \ge 0,\ k \in \overline{1,K},$$

and the middling and the variability of the value b_k equal respectively to:

$$\mathbf{M}b_k = m^k + M,\ \mathbf{D}\left(b_k\right) = 4m^k + 2M.$$

For large values of the parameter m^k, the noncentral χ^2-distribution with M degrees of freedom approximates the normal distribution with mean $\mathbf{M}b_k$ and variance $\mathbf{D}(b_k)$.

Since the components of the noise vectors $\mathbf{n}^k$ were assumed to be independent random values, the components z_i^k of the fragments of CI are statistically independent and the same conclusion can be drawn relative to random values b_k. Then the K-dimensional density of the distribution of probabilities of a vector $\mathbf{b}$ is expressed through the product of one-dimensional density of values b_k:

$$P\left(\frac{A}{B_p}\right) = \int_0^{\infty} W_{b_k}(y) \prod_{\substack{k=1 \\ k \ne p}}^{K} I_k(y)dy, \tag{13}$$

where

$$I_k(y) = \frac{e^{-m^k/2}}{2^{M/2}} \sum_{i=0}^{\infty} \frac{\left(m^k/4\right)^i}{i!\Gamma\left(i + M/2\right)} \int_y^{\infty} x^{i-1+M/2} e^{-x/2} dx. \tag{14}$$

Considering the definition of incomplete Γ-function $\Gamma(\alpha, y) = \int_y^{\infty} x^{\alpha-1} e^{-x} dx$, and the decomposition

$$\Gamma(\alpha, y) = \Gamma(\alpha) - e^{-y} y^{\alpha} \sum_{n=0}^{\infty} \frac{y^n \Gamma(\alpha)}{\Gamma(\alpha + n + 1)},$$

the expression (14) can be used as a practical form:

$$I_k(y) = 1 - e^{-\left(y+m^k\right)/2} \left(y/2\right)^{M/2} * \\ * \sum_{i=0}^{\infty} \frac{(m^k\ y/4)^i}{i!} \sum_{n=0}^{\infty} \frac{(y/2)^n}{\Gamma\left(M/2 + i + n + 1\right)}. \tag{15}$$

In order to speed up the calculation for the fragments with $m^k >> M$, let us use the non-central χ^2-distribution approximation by the normal distribution, the result after integration is:

$$I_k(y) \approx 1 - \Phi\left(\frac{y - M - m^k}{\sqrt{2M + 4m^k}}\right), \tag{16}$$

where $\Phi(x) = \left(2\pi\right)^{-1/2} \int_{-\infty}^{x} \exp\left(-y^2/2\right) dy$ is the probability integral.

In practice, the critical function generally uses centered and normalized RI and fragments of CI. Furthermore, the energies of the RI and the fragments of CI are artificially aligned, so the quadratic difference algorithm is equivalent to the correlation algorithm.

Let us study the statistical characteristics of the centered and rationed fragment of CI.

Let's fix the fragment number, and then to shorten the record, we'll drop it.

Based on the additive model of the fragment (11), enter the following vectors:

$$\mathbf{y} = \left(\hat{y}_1/\sigma_1,\ldots,\hat{y}_M/\sigma_M\right),\ \mathbf{a} = \left(\hat{a}_1/\sigma_1\ldots,\hat{a}_M/\sigma_M\right),$$
$$\mathbf{n} = \left(\hat{n}_1/\sigma_1,\ldots,\hat{n}_M/\sigma_M\right),\ \mathbf{e} = \left(\hat{e}_1/\sigma_1,\ldots,\hat{e}_M/\sigma_M\right).$$

Then let us write down the CI model as:

$$\mathbf{y} = \mathbf{a} + \mathbf{n},\ n_i \in \mathbf{N}(0,1),\ i \in \overline{1,M}.$$

Let us study the statistical characteristics of the random value $\xi_i=(y_i-\bar{y})/\zeta$,

where $\bar{y}=\frac{1}{M}\sum_{i=1}^{M}y_i$; $\zeta=\left(\frac{1}{M}\sum_{i=1}^{M}(y_i-\bar{y})^2\right)^{1/2}$.

We need to consider that random values y_i are independent and have a normal distribution with a single variance.

Then the random value is $M\zeta^2\in\chi^2(m,M)$,

where $m=\sum_{i=1}^{M}(a_i-\bar{a})^2$, $\bar{a}=\frac{1}{M}\sum_{i=1}^{M}a_i$.

As a result:

$$W_\zeta(x)=\frac{M^{M/2}x^{M-1}e^{-(Mx^2+m)/2}}{2^{M/2-1}}\sum_{j=0}^{\infty}\frac{(mMx^2/4)^j}{j!\Gamma(j+M/2)},\ x\geq 0.$$

Since the random variables y_i and ζ are independent, applying the formula for probability density quotient from division z_i by ζ [11]:

$$W_{\xi_i}(x)=\int_{-\infty}^{\infty}W_\zeta(u)W_{z_i}(ux)|u|du,$$

we obtain:

$$W_{\xi_i}(x)=\frac{e^{-(m+\alpha^2)/2}M^{M/2}}{\sqrt{2\pi}\times 2^{M/2-1}}*\sum_{j=0}^{\infty}\frac{(mM/4)^j}{j!\Gamma(j+M/2)}\int_0^\infty \exp\left[-\frac{u^2(x^2+M)-2u\alpha x}{2}\right]u^{M+2j}du, \quad (17)$$

where $\alpha=a_i-\bar{a}$.

Using the integral representation for the function of the parabolic cylinder [12]:

$$D_\nu(z)=\frac{e^{-z^2/4}}{\Gamma(-\nu)}\int_0^\infty \exp(-zt-t^2/2)t^{-\nu-1}dt,\ \mathrm{Re}\nu<0,$$

let us reduce (17) to:

$$W_{\xi_i}(x)=\frac{\exp\left[-\frac{m+\alpha^2}{2}+\frac{\alpha^2x^2}{4(x^2+M)}\right]}{\sqrt{\pi M}\times 2^{(M-1)/2}}*\sum_{j=0}^{\infty}\frac{(m/4)^j\Gamma(M+1+2j)D_{-M-1-2j}\left(-\frac{\alpha x}{\sqrt{x^2+M}}\right)}{j!\Gamma(j+M/2)(1+x^2/M)^{(M+1)/2+j}}, \quad (18)$$

where $D_\nu(x)$ is the function of the parabolic cylinder [12] To perform practical calculations with the function $D_\nu(x)$, it is convenient to present it as[12]:

$$D_\nu(x)=2^{\nu/2}e^{-x^2/4}\hat{D}_\nu(x),$$

where

$$\hat{D}_\nu(x)=\frac{\Gamma(1/2)}{\Gamma[(1-\nu)/2]}*\Phi\left(-\frac{\nu}{2},\frac{1}{2};\frac{x^2}{2}\right)+\frac{x}{\sqrt{2}}\frac{\Gamma(-1/2)}{\Gamma(-\nu/2)}\Phi\left(\frac{1-\nu}{2},\frac{3}{2};\frac{x^2}{2}\right), \quad (19)$$

where $\Phi(h,c;x)=\sum_{i=0}^{\infty}\frac{(h)_i x^i}{(c)_i}$ – is the confluent hypergeometric function;

$(c)_i=c(c+1)\ldots(c+i-1)=\Gamma(c+i)/\Gamma(c)$, $(c)_0=1$.

Then the probability density (18) is described by the expression:

$$W_{\xi_i}(x)=\frac{e^{-(m+\alpha^2)/2}}{\sqrt{\pi M}\times 2^M[\varphi(x)]^{(M+1)/2}}*\sum_{j=0}^{\infty}\left[\frac{m}{8\varphi(x)}\right]^j\frac{\Gamma(M+1+2j)}{j!\Gamma(j+M/2)}\hat{D}_{-n-1-2j}\left(-\frac{\alpha x}{\sqrt{M\varphi(x)}}\right), \quad (20)$$

where

$$\varphi(x)=1+x^2/M. \quad (21)$$

It can be demonstrated that he initial moments of $2l$-th and $(2l+1)$-th type of the distribution (17) are determined by the relations:

$$M\xi_i^{2l}=e^{-(m+\alpha^2)/2}M^l\sum_{j=0}^{\infty}\frac{(m/2)^j}{j!\prod_{k=1}^{l}(M/2+j-k)}*\sum_{i=0}^{\infty}\frac{(\alpha^2/2)^i}{i!}(i+1/2)_l, \quad (22)$$

$$M\xi_i^{2l+1}=\frac{\alpha e^{-(m+\alpha^2)/2}M^{l+1/2}}{\sqrt{2}}*\sum_{j=0}^{\infty}\frac{(m/2)^j\Gamma(j+M/2-l-1/2)}{j!\Gamma(j+M/2)}\sum_{i=0}^{\infty}\frac{(\alpha^2/2)^i}{i!}(i+3/2)_l. \quad (23)$$

The following expressions for the mean and variance of the random value ξ_i results from(22) and (23):

$$\mathbf{M}\xi_i=\alpha\sqrt{M/2}\times e^{-m/2}\sum_{j=0}^{\infty}\frac{(m/2)^j\Gamma(j+M/2-1/2)}{j!\Gamma(j+M/2)}, \quad (24)$$

$$\mathbf{D}\xi_i=\frac{M}{2}(1+\alpha^2)e^{-m/2}\sum_{j=0}^{\infty}\frac{(m/2)^j}{j!(j+M/2-1)}-(\mathbf{M}\xi_i)^2. \quad (25)$$

Study of asymptotic properties of distribution (17) based on the theory of asymptotic evaluation [13] resulted in following:
1) if $M \to \infty$, m<<M

$$\mathbf{W}_{\xi_i} \sim \mathbf{N}(\alpha,1), \quad (26)$$

i.e. the distribution is asymptotically oriented towards a normal with an average value α and a unit variance;
2) if $M \to \infty$, m<<M , the expressions (18), (19) for mean and variance take the form of:

$$\mathbf{M}\xi_i \sim \alpha\sqrt{M/m},\ \mathbf{D}\xi_i \sim M/m. \quad (27)$$

If the vector $\mathbf{a = 0}$, i.e. $\{\xi_1,\dots,\xi_M\}$ – is a repeated sample from the standard normal distribution, then $m=0$ and the distribution (17) move to the Student distribution with M degrees of freedom:

$$W_{\xi_i}(x)=\frac{\Gamma\left((M+1)/2\right)}{\sqrt{\pi M}\Gamma(M/2)\left(1+x^2/M\right)^{(M+1)/2}},$$

moreover, from (25) it follows that $\mathbf{D}\xi_i = M/(M-2)$, M>2.

It can be shown that the density of the distribution (17) is invariant with respect to linear transformations of the observation vector $\hat{\mathbf{y}}$.

The picking algorithm has the following DF:

$$b_k=\sum_{i=1}^{M}\xi_i^k\tilde{e}_i^k,\ \tilde{e}_i^k=\left(e_i^k-\bar{e}^k\right)\left[\frac{1}{M}\sum_{i=1}^{M}\left(e_i^k-\bar{e}^k\right)^2\right]^{-1/2},$$
$$\bar{e}^k=\frac{1}{M}\sum_{i=1}^{M}e_i^k. \quad (28)$$

Calculations have shown that probability density (17) is well approximated by normal density with parameters $\mathbf{M}\xi_i^k, \sqrt{\mathbf{D}\xi_i^k}$, defined by formulae (24), (25). Using this approximation, assuming that $\xi_i^k \left(i\in\overline{1,M}\right)$ are the independent random values, we obtain:

$$b_k \in \mathbf{N}(\gamma_k, s_k),\ \gamma_k=\sum_{i=1}^{M}\tilde{e}_i^k\mathbf{M}\xi_i^k,$$
$$s_k=\left[\sum_{i=1}^{M}\left(\tilde{e}_i^k\right)^2\mathbf{D}\xi_i^k\right]^{1/2},\ k\in\overline{1,K}.$$

Then according to (4) the statistics b_k have a normal distribution with mean $\gamma_k=\sum_{i=0}^{M}\tilde{e}_i\mathbf{M}\xi_i^k$ and variance $s_k^2=\sum_{i=1}^{M}\tilde{e}_i^2\mathbf{D}\xi_i^k$.

By substituting this probability density in formula (8), we find a ratio for the probability of DF generation, provided that the number of source fragment equals to p:

$$P\left(A/B_p\right)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-x^2/2}\prod_{\substack{k=1\\k\neq p}}^{K}\Phi\left(\frac{xs_p-\gamma_k+\gamma_p}{s_k}\right)dx. \quad (29)$$

2.2 The results of the statistical experiment on algorithms of DF generation

The object of the experiment:
1. Evaluation of the efficiency of algorithms using statistical modelling (Monte-Carlo technique);
2. Validation of formula (29) based on the comparison with statistical modelling results.

An image comparison algorithm based on sequential comparison of the RI with the $M_1 \times M_2$-sub-matrices (fragments) of the CI matrix and decision making on the extreme of the critical function, was simulated. Prior to comparison, each piece of CI was centered and normalized:

$$y_{ij}^{kl}=\frac{T_{ij}^{kl}-\hat{T}^{kl}}{\sigma_{kl}};$$
$$\hat{T}^{kl}=\frac{1}{M}\sum_{i=1}^{M_1}\sum_{j=1}^{M_2}T_{ij}^{kl}; \quad (30)$$
$$\hat{\sigma}_{kl}=\frac{1}{M}\sum_{i=1}^{M_1}\sum_{j=1}^{M_2}\left(T_{ij}^{kl}-\hat{T}^{kl}\right)^2,$$

where T_{ij}^{kl} – is the (i,j)-th element of the fragment, whose left upper corner has coordinates (k,l) in CI matrix;
$\hat{T}^{kl}, \hat{\sigma}_{kl}^2$ – is the evaluation of the mean value and variance of the reference (k,l)-th fragment;

$$M=M_1M_2;\ i\in\overline{1,M_1};\ j\in\overline{1,M_2};$$
$$k\in\overline{1,R_1};\ l\in\overline{1,R_2};\ R_1=N_1-M_1+1;$$
$$R_2=N_2-M_2+1.$$

Similar operations are carried out over RI. It is further assumed that the prior probabilities of all fragments are identical. Then in a strong scale, the optimal algorithm according to maximum likelihood criterion is a quadratic difference algorithm:

$$B(k,l)=-\frac{1}{M}\sum_{i=1}^{M_1}\sum_{j=1}^{M_2}\left(y_{ij}^{kl}-e_{ij}\right)^2, \quad (31)$$

which, because of the centrality and normalization of each fragment of CI and the similar RI property, is equivalent to a correlation:

$$B(k,l)=\frac{1}{M}\sum_{i=1}^{M_1}\sum_{j=1}^{M_2}y_{ij}^{kl}e_{ij}. \quad (32)$$

To implement the algorithm in the nominal scale as well as in order and hyperorder scales, it is necessary first to solve for each fragment the problem of synthesis of the optimal RI of the form:

$$B(k,l;e)=\min_{\mathbf{e}\in E}\sum_{i=1}^{M_1}\sum_{j=1}^{M_2}\left(y_{ij}^{kl}-e_{ij}\right)^2 , \quad (33)$$

where the set E is defined by the type of scale in which the RI is represented. The centered and normalized solution of problem (33) is received in the [7] for the nominal scale, algorithms for order and hyperorder scales. The decision is then made according to the maximum of decision function (31) or (32).

Statistical experiments has been performed for algorithms of all the types considered, i.e. repeated running of algorithms for different CI implementations, comparing the coordinate estimate $\left(\hat{i}_{op},\hat{j}_{op}\right)$, obtained at each start with the true value (coordinates of source fragment) $\left(i_{op},j_{op}\right)$, which was specified in the modelling of the CI.

If the coordinates coincide, a conclusion is drawn about the DF generation during the processing of this implementation, and an assessment of the probability of the DF generation by the ratio of the formed DF to the total number of runs of algorithms in the test series. For an algorithm in a strong scale, efficiency estimates are calculated using an expression (29).

Statistical experiment results.

CI with dimensions $N_1=N_2=16$ (Figure 1) and fixed brightness of individual zones was modelled. The brightness of the first zone varied during the tests. CI elements with the dimensions $M_1=M_2=4$ and number of zones $N=3$.

The total number of implementations processed in the series, used to calculate one probability value P_c, is N_z = 400. Algorithms, which use centered and normalized RI and CI fragments, were tested as well as those for nominal, ordinal, hyperorder and absolute scales. Information about the numerical representation of RI brightness was used only by the algorithm in a strong scale.

1	1	1	1	1	1	1	1	2	2	2	2	3	3	3	3
1	1	1	1	1	1	1	1	2	2	2	2	3	3	3	3
2	2	2	2	2	2	2	2	2	2	2	2	3	3	3	3
2	2	2	2	2	2	2	2	2	2	2	2	3	3	3	3
3	3	3	3	3	3	3	3	3	3	2	2	3	3	3	3
3	3	3	3	3	3	3	3	3	3	2	2	3	3	3	3
3	3	3	3	3	3	3	3	3	3	2	2	3	3	3	3
3	3	3	3	3	3	3	3	3	3	2	2	3	3	3	3
1	1	1	1	1	1	1	1	2	2	2	2	3	3	3	3
1	1	1	1	1	1	1	1	2	2	2	2	3	3	3	3
1	1	1	1	1	1	3	3	3	3	3	3	3	3	3	3
1	1	1	1	3	3	3	3	3	3	3	3	3	3	3	3
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
3	3	3	1	1	1	1	1	1	1	3	3	3	3	3	3
1	1	1	1	1	1	1	1	1	1	3	3	3	3	3	3

Figure 1: Modelled CI

In Figure 2 the dependence of efficiency of nominal (results have labels "+"), ordinal ("×") algorithms and algorithm in strong scale (circles) against contrast $\Delta T=T_2-T_1$ for $\sigma=3K$ and $\sigma=7K$ in case of fixed other parameters of CI and RI, is presented. On the same graph, solid fat curves show theoretic dependency graphs, drawn based on the formula(29). The differences in the efficiency of ordinal and hyperordinal algorithms have been so small that they cannot be reflected on graphs.

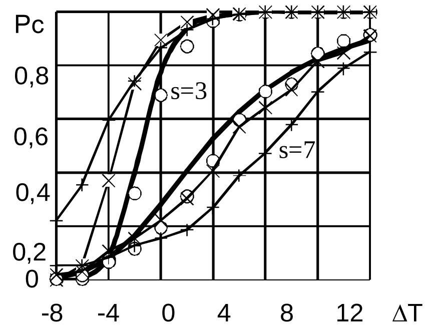

Figure 2: Dependence of P_c on ΔT

Thus, it is shown that:

1) at higher noise levels (small noise-to-signal ratios), the algorithmin a strong scale is slightly more efficient than the ordinal one, which at the same time is the most sensitive to inversion of contrast ΔT ;
2) at high noise-to-signal ratio the algorithm in a strong scale is significantly inferior to the ordinal one, and in case of a substantial inversion of the contrast, the nominal algorithm has the best performance;
3) the satisfactory correspondence between the theoretical results and the results obtained from the statistical simulation for a strong scale algorithm has been received.

3. CONCLUSION

As a result of the performed researches, it has been shown that in MP navigation systems from system positions the correlation algorithm of image comparisons is the most preferred for application, and the formation of RI must be carried out in a strong scale.

REFERENCES

1. V. Antyufeev. **Matrix radiometric correlation-extreme navigation systems for aircraft**: monographUkraine, Kharkov: KhNU V.N. Karazin, 372 p.

2. A. Sotnikov, V. Tarshyn, N. Yeromina, S. Petrov, N. Antonenko. **A method for localizing a referenceobject in a current image with several bright objects**, Eastern-European Journal of Enterprise Technologies. 2017. Vol. № 9 (87). pp. 68–74. DOI: 10.15587/1729-4061.2017.101920.

3. N.Yeromina, S. Petrov, A. Tantsiura, M. Iasechko, V. Larin. **Formation of reference images and decisionfunction in radiometric correlation-extremal navigation systems.** Eastern-European Journal ofEnterprise Technologies. — 2018. Vol.4, No.9 (94). — pp. 27—35. DOI: 10.15587/1729-4061.2018.139723.

4. O.Tymochko , A.Ttystan , V.Ushan , N.Yeromina , O.Dmitriiev , V.Mazharov , I.Padalka , I.Hannoshyna , I. Masik , A. Zazirnyi**The Synthesis of the Reference Image and Algorithms for Vehicle Navigation Systems,**JETER,8(3), 2020, pp. 853- 858. doi: 10.30534/ijeter/2020/40832020
5. Pfanzagl I. Measurement theory. - Moscow: Mir, 1976. - 248 p.
6. S. Trehubenko, L. Berkman, N.Yeromina, S. Petrov, Y. Bryzhatyi, H. Kovalov, V.Dachkovskyi, L.Mikhailova**The Operation of Detection Systems in Conditions of Contrast Decrease of Ground Objects,** JETER, 8 (1), 2020, pp. 208-212, doi: 10.30534/ijeter/2020/28812020
7. N.Yeromina, S. Petrov, A. Tarasenko, N. Antonenko, O. Semenenko, O. Pavlovskyi, M. Sliusarenko, V. Telehin**The Use of Strong and Ordinal Scales during the Synthesis of Reference Images for Vehicle Correlation-Extreme,** IJATCSE, 9 (2), 2020, pp. 2343-2349, doi: 10.30534/ijatcse/2020/218922020
8. Grenander W. **Lectures on the theory of images** : TRANS.: In 3 t. - M.: Mir. Vol. 1: image Synthesis. 1979. - 383 p.
9. Gutkin L.S. **Optimization of electronic devices by a set of quality indicators**. - M .: Owls. Radio, 1975 .-- 368 p.
10. N. Yeromina, S.Petrov, N.Antonenko, I.Vlasov, V.Kostrytsia, V. Korshenko**The Synthesis of the Optimal Reference Image Using Nominal and Hyperordinal Scales,** JETER, 8 (5), 2020, pp. 2080-2084, doi: 10.30534/ijeter/2020/98852020
11. LevinB.R. Theoreticalfoundationsofstatisticalradioengineering: In 2 t. / T. 1. - M .: Sov. Radio, 1974.- 552 p.
12. Beitman G., Erdeyi A. Higher transcendental functions. Bessel functions, parabolic cylinder functions, orthogonal polynomials: Per. from English - M.: Nauka, 1974.- 295 p.]
13. Evgrafov M.A. Asymptotic estimates and entire functions. - M .: Nauka, 1979. - 320 p.